

Cylindrical symmetry locally isometric AdS_4 spacetime

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Abstract

We present a maximally symmetric vacuum spacetime, which is locally isometric anti- de Sitter, admitting closed timelike curves appear after a definite instant of time i.e., a time-machine spacetime. The spacetime is regular, free-from curvature singularities and an $4D$ extension of the Misner space in curved spacetime with identical causality violating properties.

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1 Introduction

Anti-de Sitter space/Conformal Field Theory (AdS/CFT) conjecture has been a central paradigm for decades in theoretical physics from string theory to condensed matter physics [1]. Understanding and a fully description of the AdS space plays a crucial role in this paradaim. AdS spacetime is an exact solution for Einstein's field equations with a negative cosmological constant. The lowest-energy solution is the Ads spacetime and furthermore it was demonstrated that AdS is the unique static, asymptotically anti-de Sitter vacuum [2].

Reciprocal transformations proposed by Buchdahl is a simple way to find exact solutions for Einstein field equations in the presene of a massless scalar field. [3]. Based on symmetry we can have both spherically or cylindrically metrics as far as we can have time dependent scalar field as sourceof the static metrics. Later Wynman proposed a simple series solution for the case of massless scalar field but when the scalar field is time ependent [5]. A higher dimensional form of metric was found in [6]. Very recently Vuille found an exact solution for Einstein gravity with a non zero (positive or negative) cosmological constant in plane symmetry [7]. Another interesting class of exact solutions are ones found by Anzhong Wang with homothetic self-similarity [40]. A natural generlization of the static solutions beyound four dimensions investigated by Sarioglu and Tekin [9]. In (2009) one of us found an exact solution for Einstein gravity in the presence of an arbitrary cosmological constant and with a static scalar field as a generalization of the Buchdahl extension of the Levi-Civita metric [10]. This solution is a new cylindrically symmetric solution for a massless scalar field coupled minimally to gravity but contains cosmological constant with regularity for all real values of scalar field parameter and with a naked singularity for complex one

AdS spacetimes could be obtain by embedding in a higher dimensional flat space. This showed that for example *AdS*₃ were already embed ded into the 4*D* flat spacetime $\mathcal{R}^{2,2}$ with metric $g = -(dX_1^2 + dX_2^2) + (dX_3^2 + dX_4^2)$. This embedding is invariant under $SO(2,1)$, which is the precise isometry group of *AdS*, with confor- mal boundary. An appropriate rescaling casts the boundary in hyperboloid form in (2)*D* form, which is universal for systems supporting $SO(1,2)$ isometry group, with one dilatation and two special conformal transformations.

Some solutions of Einsteins field equations have peculiar properties called closed causal curves (CCCs). These CCCs are closed timelike curves (CTCs), closed timelike geodesics (CTGs), closed null curves (CNC) or geodesics (CNGs) etc.. In literature, there are a number of solutions of field equations that have CCCs. A sample of these are [20]- [32]. One way of classifying such causality violating solutions would be to categorize the spacetime as either eternal time-machine in which CTCs always exist,(e.g. [20–22]. The second one being the true time-machine spacetime, where CTCs appears after a certain instant of time. In this direction Amos Ori deserves the special mention [33]- [37]. We note that the eternal time-machine spacetimes are unrealistic models for a putative time- machine. A workable model of a time-machine must

be a spacetime where CTCs appear at a definite instant of time. Among the true time-machine spacetime, we mention two: the first being the Oris vacuum core which is represented by a vacuum metric locally isometric to plane wave [34] and second, the Misner space in 2D [35]. The Misner space is interesting in the context of CTCs as it is a prime example of a spacetime where CTCs evolve from causally well-behaved initial conditions.

The metric for the Misner space in 2D [35]

$$ds^2 = -2 dt dx - t dx^2 \quad . \quad (1.1)$$

where $-\infty < t < \infty$ but the co-ordinate x periodic. The metric (1.1) is regular everywhere as $\det g = -1$ including at $t = 0$. The curves $t = t_0$, where t_0 is a constant, are closed since x is periodic. The curves $t < 0$ are spacelike, but $t > 0$ are timelike and the null curves $t = t_0 = 0$ form the chronology horizon. The second type of curves, namely, $t = t_0 > 0$ are closed timelike curves (CTCs).

The Einstein Field equation in electrovacuum is given by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \quad or \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \quad (1.2)$$

where $\mu, \nu \in (1, 2, 3, 4)$. For conformally flat spacetime the Weyl tensor defined by

$$C_{\alpha\beta\mu\nu} = R_{\alpha\beta\mu\nu} + \frac{1}{2} \left(-R_{\alpha\mu} g_{\beta\nu} + R_{\alpha\nu} g_{\beta\mu} + R_{\beta\mu} g_{\alpha\nu} - R_{\beta\nu} g_{\alpha\mu} + \frac{1}{3} (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}) R \right)$$

vanishes, the Riemann tensor $R_{\mu\nu\rho\sigma}$ can be expressed in terms of metric tensor $g_{\mu\nu}$ by

$$R_{\mu\nu\rho\sigma} = k (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \quad . \quad (1.3)$$

AdS_4 space has simply the non trivial topology $\mathcal{R} \times \mathcal{H}^3$ is the hyperbolic metric in polar coordinates. Another interesting dual spacetime example is the so called AdS soliton, which its topology is $\mathcal{R} \times \mathcal{B} \times \mathcal{T}$.

In this letter, we attempted to write down an axially symmetric form of anti-de Sitter space where closed timelike curves (CTCs) appear after a certain instant of time a time-machine spacetime. The spacetime is an extension of four-dimensional Misner space and the time- machine behaviour is carried over to the AdS space. We note that there has been earlier attempt at constructing a Misner-like AdS spacetime by Li [38] having the line element

$$ds^2 = -(dt - t d\chi)^2 + \alpha^2 d\chi^2 + (dy - y d\chi)^2 + (dz - z d\chi)^2 \quad (1.4)$$

In (1.4), there are no CTCs when $t^2 < \alpha^2 + y^2 + z^2$, but CTCs appear in the region with $t^2 > \alpha^2 + y^2 + z^2$. However, this spacetime, as in the case of the Misner space, is multiply- connected, and we attempt to overcome this problem by constructing a Misner-like, AdS_4 spacetime which is simply-connected.

2 Review of exact solutions with cosmological constant in cylindrical coordinates

We begin with a general cylindrically symmetric metric in Weyl coordinates (t, r, φ, z) , in natural units

$$ds^2 = -e^{u(r)} dt^2 + dr^2 + e^{v(r)} d\varphi^2 + e^{w(r)} dz^2 \quad (2.5)$$

Field equation for a massless minimally coupled scalar field in the presence of a cosmological constant term Λ is reading as:

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = \phi_{;\mu} \phi_{;\nu} \quad (2.6)$$

Metric functions are given by $u_i = \{u(r), v(r), w(r)\}$, $\phi \equiv \phi(r)$, $\dot{f} = \frac{df}{dr}$. The system of the field equations are given by the following:

$$2u_i'' + u_i' \sum_{j=1}^3 u_j' - 4\Lambda = 0, i = \{1, 2, 3\} \quad (2.7)$$

$$2 \sum_{j=1}^3 u_j'' + \sum_{j=1}^3 u_j'^2 - 4\Lambda = 4\phi'^2 \quad (2.8)$$

It was showed that the system has different exact solutions as follows:

- solution with $u(r) = \text{constant}, w(r) = \text{constant}, \phi' = 0$

The metric reads:

$$ds^2 = -dt^2 + dr^2 + (ar)^2 d\varphi^2 + dz^2$$

By applying change of coordinates:

$$\tilde{\varphi} = a\varphi$$

we can find

$$ds^2 = -dt^2 + dr^2 + r^2 d\tilde{\varphi}^2 + dz^2$$

Which obviously is flat space (locally) in cylindrical Weyl coordinates and the conical parameter a related to the gravitational mass per unit length of the spacetime, η [11]

$$a = 1 - 4\eta, \quad 0 < a < 1.$$

- solution with $u(r) = v(r) = w(r) \neq \text{constant}, \phi' \neq 0$

In this case we obtain the following exact solution:

$$ds^2 = dr^2 + e^{-2\sqrt{\frac{\Lambda}{3}}r} (\xi^2 e^{2\sqrt{3\Lambda}r} + 1)^{2/3} (-dt^2 + d\varphi^2 + dz^2) \quad (2.9)$$

It was reported in [10] as an exact solution which contains two parameter, Cosmological constant Λ and scalar field susceptibility parameter ξ , permits one time-like Killing fields $\frac{\partial}{\partial t}$, three space-like Killing fields, which span a Euclidean group $G(3)$ (see [40] for cosmological forms and see [12]- [19] for extensions in modified gravity theories).

By computing the Kretschmann scalar we find:

- If $\xi^2 > 0$ i.e. the scalar field's parameter is a real constant, no naked singularity at $r = r_0$.
- If $\xi \in \mathcal{C}$ has naked singularity

This solution recovers safely LCA and Buchdahl solutions, in the limit of the solution with $\Lambda \neq 0, \phi = \text{constant}$ (LCA family):

$$ds^2 = dr^2 + e^{\pm 2\sqrt{\frac{\Lambda}{3}}r}(-dt^2 + d\varphi^2 + dz^2) \quad (2.10)$$

is a member of general LCA family and in limit $\Lambda = 0, \phi \neq \text{constant}$ reduces to the Buchdahl family solution:

$$ds^2 = dr^2 + \left[\frac{3}{2}(c_1 r + c_2)\right]^{\frac{2}{3}}(-dt^2 + d\varphi^2 + dz^2) \quad (2.11)$$

3 Analysis of the spacetime

Cylindrical symmetric metric

$$ds^2 = dr^2 + e^{2\alpha r} (dz^2 - \beta t r d\phi^2 - \beta r dt d\phi - \beta t dr d\phi) \quad (3.12)$$

where ϕ co-ordinate is periodic $0 \leq \phi \leq 2\pi$, and α is an integer and β is real. We have used co-ordinates $x^0 = t$, $x^1 = r$, $x^2 = \phi$ and $x^3 = z$. The ranges of the other co-ordinates are $t, z \in (-\infty, \infty)$ and $0 \leq r < \infty$. The metric has signature $(-, +, +, +)$ and the determinant of the corresponding metric tensor $g_{\mu\nu}$, $\det g = -\frac{1}{4}r^2 e^{6\alpha r}$, vanishes at $r = 0$. We note that for spacetime (3.12), Ricci scalar $R = -12\alpha^2$ and $k = -\alpha^2$. The Einstein tensor is the diagonal form given by

$$G^\mu_\mu = 3\alpha^2 = -\Lambda \delta^\mu_\mu \quad (3.13)$$

Consider closed orbits of constant $t = t_0$, $r = r_0$ and $z = z_0$, the line element (3.12) reduces to (1) D form

$$ds^2 = -\beta t r d\phi^2 \quad (3.14)$$

These orbits are null curves for $t = t_0 = 0$, spacelike throughout for $t = t_0 < 0$, but become timelike for $t = t_0 > 0$, which indicates the presence of CTCs. Here we choose $r = r_0 > 0$, a constant and $\beta > 0$ such that the above curves are timelike. Hence CTCs form at a specific instant of time satisfying $t = t_0 > 0$.

It is crucial to have analysis that the above CTCs evolve from an spacelike $t = \text{constant}$ hypersurface and thus t is a time coordinate [34]. This can be ascertained by calculating the norm of the vector $\nabla_\mu t$ or by determining the sign of the component g^{tt} in the metric tensor $g^{\mu\nu}$ [34]. We find from metric (3.12) that

$$g^{tt} = \frac{t (4 r e^{-2\alpha r} + \beta t)}{\beta r^2} . \quad (3.15)$$

A hypersurface $t = \text{constant}$ is spacelike provided $g^{tt} < 0$ for $t = t_0 < 0$, but become timelike for $t = t_0 > 0$ provided $g^{tt} > 0$. The hypersurface $t = \text{constant}$ have a true singularity at $r = 0$. Our analysis throughout the paper is restricted to the range $r > 0$. Thus we consider $r = r_0 > 0$, a constant and β is sufficiently small positive number (ensuring that the bracket term (3.15) is positive for $t < 0$) such that $t = \text{constant}$ hypersurface is spacelike. Thus $t = \text{constant}$ spacelike hypersurface can be chosen as initial conditions over which the initial data may be specified. There is a Cauchy horizon for $t = t_0 = 0$ called chronology horizon which separates the causal and non-causal of the spacetime. Hence the spacetime evolves from a Partial Cauchy hypersurface in a causally well-behaved manner, upto a moment, i.e., a null hypersurface $t = t_0 = 0$ and CTCs evolve from this spacelike Cauchy hypersurface at a specific instant of time.

4 Axially symmetry of the spacetime

Consider the following Killing vector $\eta = \partial_\phi$ which has the normal form

$$\eta^\mu = (0, 0, 1, 0) . \quad (4.16)$$

Its co-vector form given by

$$\eta_\mu = -\beta e^{2\alpha r} \left(\frac{r}{2}, \frac{t}{2}, r t, 0 \right) \quad (4.17)$$

Killing vector (4.16) satisfies the Killing equation $\eta_{\mu;\nu} + \eta_{\nu;\mu} = 0$. For cyclically symmetric metric, the norm $\eta_\mu \eta^\mu$ of the Killing vector is spacelike, closed orbits. We note that

$$\eta^\mu \eta_\mu = -\beta t r e^{2\alpha r} , \quad (4.18)$$

which is spacelike for $t < 0$, closed orbits (as β is positive real nubmer). For axially symmetric, the norm (4.18) must vanishes on the axis $r = 0$ [39–42] is satisfied. The regularity-condition, namely that in the limit of rotation axis ($r \rightarrow 0$) one must have [39–42]

$$\frac{(\nabla_a(\eta_\mu \eta^\mu))(\nabla^a(\eta_\mu \eta^\mu))}{4 \eta_\mu \eta^\mu} \rightarrow 1 \quad (4.19)$$

holds for the metric defined by (3.12) for $t < 0$. This ensures the validity of imposing the 2π periodicity on the ϕ co-ordinate. Hence the spacetime (3.12) is regular on the axis and cylindrially symmetric with r is the true radial coordinate.

In spacetime (3.12), we have done transformation $t \rightarrow \frac{t}{\beta r}$

$$ds^2 = dr^2 + e^{2\alpha r} (dz^2 - t d\phi^2 - dt d\phi) \quad (4.20)$$

Still a co-ordinate singularity $t \rightarrow \pm\infty$. To remove this singularity, we have done transformation

$$\phi \rightarrow \phi - \frac{1}{2} \ln t \quad (4.21)$$

followed by

$$t \rightarrow (\phi^2 - t^2) \quad \text{and} \quad \phi \rightarrow \tanh^{-1}\left(\frac{\phi}{t}\right) \quad (4.22)$$

We get the diagonal form of the metric (4.20)

$$ds^2 = dr^2 + e^{2\alpha r} (dz^2 + d\phi^2 - dt^2) \quad (4.23)$$

Substituting $\alpha = \pm \sqrt{-\frac{\Lambda}{3}}$ using (3.13) into (4.23), we get the form [43], [10]

$$ds^2 = dr^2 + e^{\pm 2\sqrt{-\frac{\Lambda}{3}}r} (dz^2 + d\phi^2 - dt^2) \quad (4.24)$$

Spacetime with metric (4.24) constitutes a part of the anti-de Sitter spacetime with the metric written in the horospherical coordinates. This follows from the following transformation into the metric (4.24)

$$r = \mp \sqrt{-\frac{3}{\Lambda}} \ln\left(\sqrt{-\frac{\Lambda}{3}} x\right)$$

One get conformally flat standard form of locally isometric AdS_4 space [43]

$$ds^2 = \frac{3}{(-\Lambda)x^2} (-dt^2 + dx^2 + d\phi^2 + dz^2) \quad (4.25)$$

where $\Lambda < 0$ and one of the co-ordinate ϕ being periodic. Solution (4.25) with the '+' sign covers that part of the anti-de Sitter hyperboloid where $x \in [\sqrt{\frac{3}{(-\Lambda)}}, \infty)$ while the '-' branch is valid for $x \in [\sqrt{\frac{3}{(-\Lambda)}}, 0)$. However, metric (3.12) is axially symmetric and $r = 0$ represents the axis of rotation. The spacetime (3.12) is free-from curvature singularities and the curvature invariant (Kretschmann scalar K)

$$K = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 24\alpha^2$$

which is a constant.

Consider a null vector k_μ for metric (3.12) whose normal form given by

$$k_\mu = \left(0, 0, \frac{1}{2}, 0\right) \quad (4.26)$$

The null vector (4.26) satisfy the geodesics equation

$$k_{\mu;\nu} k^\nu = 0 \quad (4.27)$$

Using the null vector (4.26) we have calculated the *Optical* scalar [42], the *expansion*, the *twist* (rotation) and the *shear* and they are

$$\theta = \frac{1}{2} k^\mu_{;\mu} = 0 \quad (4.28)$$

$$\omega^2 = \frac{1}{2} k_{[\mu;\nu]} k^{\mu;\nu} = 0 \quad (4.29)$$

$$\sigma \bar{\sigma} = \frac{1}{2} k_{(\mu;\nu)} k^{\mu;\nu} - \theta^2 = 0 \quad . \quad (4.30)$$

Hence the spacetime (3.12) is non-diverging, have shear-free null geodesics congruence.

5 Fermi-Robertson-Walker (FRW) flat space

It is interesting to note that by doing a complex transformation into the spacetime (4.24)

$$r \rightarrow i\tau \quad \text{and} \quad t \rightarrow iu \quad (5.31)$$

one get 4D de-Sitter spacetime with $\Lambda > 0$ as

$$ds^2 = -d\tau^2 + e^{\mp 2\sqrt{\frac{\Lambda}{3}}\tau} (d\phi^2 + dz^2 + du^2) \quad . \quad (5.32)$$

The spacetime (5.32) can be written as

$$ds^2 = -d\tau^2 + f^2(\tau) (d\phi^2 + dz^2 + du^2) \quad , \quad (5.33)$$

where

$$f(\tau) = e^{\mp H_0 \tau} \quad \text{and} \quad H_0 = \sqrt{\frac{\Lambda}{3}} \quad . \quad (5.34)$$

The spacetime (5.33) with (5.34) is the Fermi-Robertson-Walker (FRW) cosmological model in (τ, u, ϕ, z) . The metric (5.33) is increasing with increasing time due to exponential factor in case of '+', hence the model is used as expanding universe. For expanding universe, transforming $t_e \rightarrow -H_0^{-1} e^{-H_0 t}$ into the metric (5.33) one get

$$ds^2 = \frac{1}{H_0^2 t_e^2} (-dt_e^2 + du^2 + d\phi^2 + dz^2) \quad , \quad (5.35)$$

where t_e runs from $-\infty$ in the far past to 0 in the far future and the Hubble parameter is given by $H = \frac{\dot{t}}{t} = H_0$.

While the model (5.33) is used as contracting universe in case of '-' sign as time increases. Hence for contracting universe, transforming $t_c \rightarrow H_0^{-1} e^{H_0 t}$ into the metric (5.33) one get

$$ds^2 = \frac{1}{H_0^2 t_c^2} (-dt_c^2 + du^2 + d\phi^2 + dz^2) \quad , \quad (5.36)$$

where $t_c \in [0, \infty]$ and the Hubble parameter is given by $H = \frac{\dot{t}}{t} = -H_0$.

6 Conclusion

We have presented cylindrical symmetric model, an extension of the Misner space in curved spacetime, admitting closed timelike curves. These curves evolves from a cauchy spacelike hypersurface after a certian instant of time and hence can be used as Time-Machine model. Our primary motivation in this paper is to write down a metric for a spacetime that incorporates the Misner space and its causality violating properties and to classify it. The solution is an axially symmetric metric (5) which serves as a model of a Time-Machine spacetime in the sense that CTCs appear at a definite instant. We also note that the spacetime represented contains the symmetry axis rendering it simply-connected. In contrast, the Misner space is multiply-connected.

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